Current Topics in Biophysics 2003, 27(1-2), 27-36

DIMENSIONAL COMPLEXITY OF POSTUROGRAPHIC SIGNALS: II. INFLUENCE OF WINDOW WIDTH ON DIMENSIONAL COMPLEXITY ESTIMATION

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To properly estimate dimensional complexity (DC) from a time series some requirements should be met as to signal recording. Moreover, some parameters necessary for reliable reconstruction of chaotic character of the series should be assessed. In this paper, we calculated the influence of an embedding parameter, window width (*W*), on dimensional complexity (DC) of posturographic signal. To this aim we used posturographic signals from 32 young healthy participants. Our results indicate that no clear value of *W* can be determined because plateau-segment in the plot DC(*W*) was not found. For further analysis, the values of *W*=0.2-1s seem to be suitable for investigations of postural reflexes and the values of *W*=1-10s for slow movement of center-of-mass examination.

INTRODUCTION

Posturography is a method enabling quantitative evaluation of state and efficiency of human balance system. In this method, location of pressure center of the body is measured and registered under different experimental conditions. Usually, this signal consists of 2 dimensions: anterio-posterior, (AP) and lateral (LAT) which as a rule are analyzed separately. Thus far, posturography has not commonly been accepted as a useful clinical diagnostic method because of insufficient sensitivity and accuracy of usual parameterization of the signal (e.g. sway path, sway area, Romberg quotient) (Motta, Spano, Neri, Schillaci, Corteloni, Andermarcher, Gamberini & Rizolli, 1991; Prieto, Myklebust, Hoffmann, Lovett & Myklebust, 1996). However, it seems to be a valuable research method enabling better understanding of human balance system and some its disturbances.

In our previous paper (Michalak & Jaśkowski, 2002) we calculated the dimensional complexity (DC) of posturographic signal, which is formally defined as (Pritchard & Duke, 1995):

$$d \propto \lim_{r\to\infty}(\log M / \log r),$$

where d – stands for DC, r – describes the object size in a single dimension, M – is the "bulk" of the object and the symbol $\propto$ stands for "is proportional to".

To apply this definition to analysis of 1-dimensional time series one has to specify what M and r mean. Function $M(r)$ in such a situation could be defined as the number of pairs of points, Euclidean distance of which being less or equal to r. "Point" should not be understood as a single value of the signal but as a point in an m-dimensional phase space, its coordinates constitute m consecutive, equally distant samples of the signal. Parameter m stands for so-called embedding dimension (ED). In order to correctly estimate DC, ED has to be large enough. Its value should be $m > 2 \cdot d_2 + 1$, where d_2 is the real dimension of the object. For example, let's consider a sequence of 100 samples x_i. If $m = 4$ and lag or delay time $L = 2$ (number of samples between components of each state vector), then we can form $n = 94$ points in the 4-dimensional space state: (x_1, x_3, x_5, x_7), (x_2, x_4, x_6, x_8), (x_3, x_5, x_7, x_9), ..., (x_{94}, x_{96}, x_{98}, x_{100}). $N = n(n-1)/2 = 94 \cdot 93/2 = 4371$ Euclidean distances can be calculated between these points. The number of distances smaller than r increases with r. Initially it increases exponentially. It was proved that the exponent of this relation is equal to DC (Pritchard & Duke, 1995). If $M(r)$ is plotted in the log-log scale the segment with exponential scaling will be approximately a straight line. It is so-called linear scaling region (LSR). DC can be estimated as the slope of LSR. In a graph displaying derivative of log M as a function of log r, LSR is viewed as an approximately horizontal straight line. Function log $M(r)$ vs log r is called Correlation Integral.

Parameters m and L determine window width, $W = (m-1)\cdot L$. W is the distance between ends of a signal fragment forming a single point in the state space. It is very important to choose properly the width of W. When W is too wide, ends of a vector will be virtually unrelated and the structure of the reconstructed attractor will be distorted. Estimated DC will be too large in such a case (see Fig.1: W = 80-160 pts). When it is too narrow, two consecutive co-ordinates of a point in the state space will have nearly the same values. In such a situation, all points of attractor will be distributed along the main diagonal of the state space and the structure of attractor will be invisible. DC will be underestimated in such a case (see fig.1: W = 1-10pts).

Thus W is, beside the minimal recording time and minimal sampling frequency, one of the most important parameters determining preciseness of DC calculations. So far, no systematic investigations were undertaken to check influence of this factor on estimated DC. This paper is aimed to determine the proper window width W for DC calculation. Because many lacunarities have been observed in the Correalation Integrals (log $C(r)$' vs log r) of posturographic signals and thus the LSR has often been difficult to determine, we used Takens-Ellner algorithm for DC calculation. This algorithm avoids problems of lacunarities and lack of LSR. Detailed description of this algorithm can be found elsewhere (Ellner, 1988; Michalak & Jaśkowski, 2002).

METHODS

Subjects

Detailed description of data acquisition and selection of participants was presented elsewhere (Michalak & Jaśkowski, 2002). Briefly, posturographic signals were registered from 32 healthy persons (18-50 years). 130-s long signals were registered with sampling frequency of 200 Hz twice, with open eyes (EO) and with closed eyes (EC). Altogether, 4 kinds of signals were analyzed separately: EO-AP, EO-LAT, EC-AP and EC-LAT. First 10-s segment of each time series was considered as stabilization time and rejected from further analysis. Thus the effective interval for calculations was 120 s.

Computational experiments

In order to estimate a proper window width for DC calculations, two computational experiments were performed:

I. Estimating autocorrelation times of analyzed signals.
II. Estimating DCs of posturographic signals using Takens-Ellner algorithm for values of W from 0.25 to 24 s and looking for a plateau segment on the graph DC(W). This segment determines the range of accepted values of W for DC calculations. Next, the differences between original and shuffled signals (DF = DC_{shuff}-DC_{orig}) and its ratios (RA = DC_{shuffl} / DC_{orig}) will be presented. As is shown in Fig. 1 for Lorenz attractor, maxima of these relations correlate approximately with the end of plateau segment.

Estimating the autocorrelation times

Autocorrelation time τ is often used to estimate W for DC calculations. It is defined as a time for which autocorrelation function of a signal decreases e-times. The first zero of autocorrelation function or its first minimum is also sometimes used. It is commonly assumed that for DC calculations the window width should be approximately $W = \tau \div 4\tau$. Empirically optimized W is as a rule the value for which the widest linear scaling region in the Correlation Integral plot is observed. The majority of signals (e.g. Lorenz Attractor, Henon equation) possess the optimal W just in the range of $W = \tau \div 4\tau$.

In order to determine how autocorrelation time depends on sequence length, autocorrelation time was calculated for sequence lengths of 10 ÷ 120 s. Original signals EO-AP, EO-LAT, EC-AP and EC-LAT were analyzed separately.

Calculation DC for different values of W

If W gradually increases then estimated DC of a signal generated by a chaotic system (e.g. Lorenz attractor) increases initially from 1 to the value of its dimensional complexity. Next, a plateau segment is observed. Finally, when W becomes large, further increase of DC is again observed. The relation DC(W) for Lorenz signal is presented in Fig.1.

One can observe that estimated DC of a chaotic signal was constant for some window widths from a given range. In Fig.1, this is the range of about W = 10 ÷ 40 pts. Moreover, difference between DCs for original and shuffled signals (DF) and its ratio (RA) decreases for large W. Approximately at the end of this range the DF(W) and RA(W) reach their maxima.

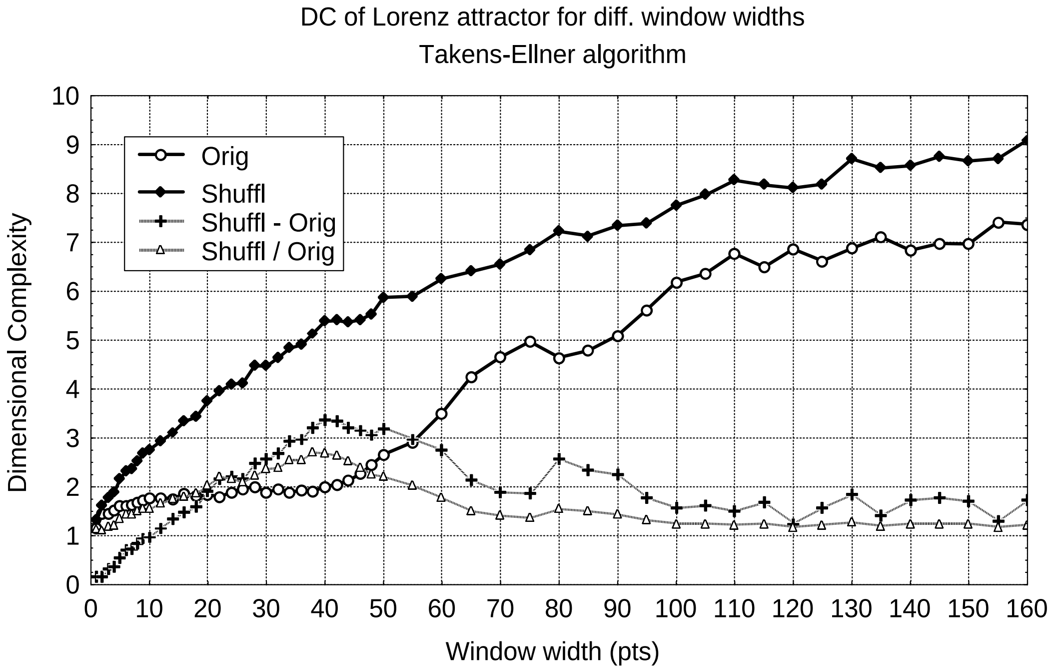


Fig. 1. Dimensional complexity of Lorenz attractor calculated by means of Takens-Ellner algorithm for different window widths. Plateau in the region of W=10-40pts determines the proper window width for DC calculations.

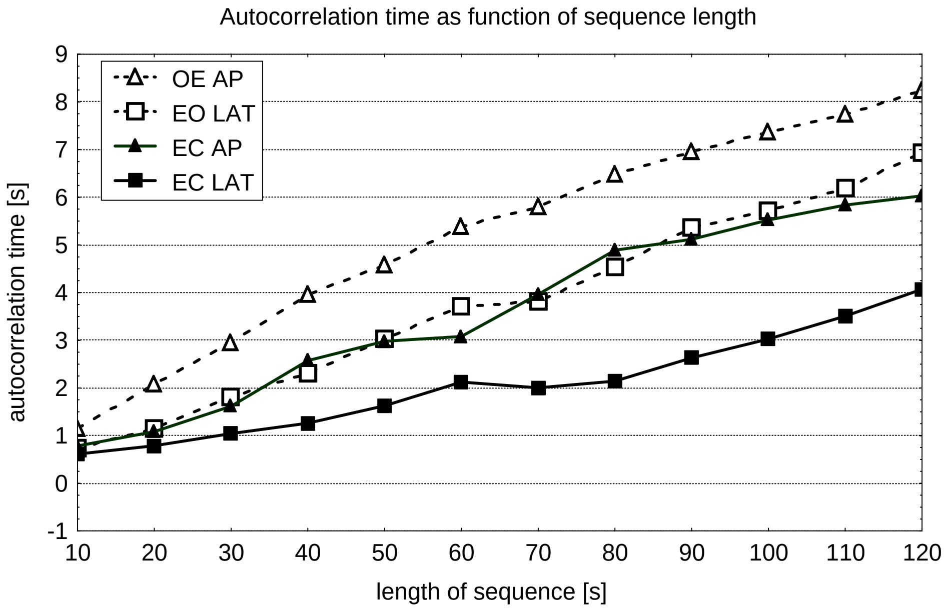


Fig. 2. Autocorrelation times for different lengths of posturographic series. EO - open eyes, EC - closed eyes, AP - anterio-posterior direction, LAT - lateral direction of movement.

In our experiment, we attempted to find such a plateau for posturographic signals and we analyzed the differences and ratios between DC of the original and shuffled signals for window widths $W = 0.25 \div 24$ s. To this end, DCs of 64 (32 for open eyes and 32 for closed eyes) 120-s long posturographic series were calculated using Takens-Ellner algorithm. The series were embedded into the state space of $m = 26$ dimensions. As $W = (m - 1)\cdot L$, the smallest window calculated for $L = 1$ gives the width of 25 signal points. For sampling frequency of 200 Hz it amounts to the width of $W = 0.125$ s. Thus, in order to obtain $W = 0.25$ s, 0.5 s, 0.75 s, ... following values of $L = 2, 4, 6, ...$ were chosen, respectively. The embedding dimension $m = 26$ seems also sufficiently large bearing in mind the rule: $m > 2d_2 + 1$ (m is the embedding dimension and d_2 - real dimension of the object). Shuffled series were constructed using so-called Surrogate-2 algorithm. Details of the algorithm used in this paper were presented elsewhere (Michalak & Jaśkowski, 2002; Rapp, Albano, Schmah & Farwell, 1993; Rapp, Albano, Zimmerman & Jimenez-Montano, 1994).

RESULTS

Autocorrelation time

Mean autocorrelation times for various sequence lengths are depicted in Fig.2.

As it is seen, autocorrelation times were not constant for different sequence lengths as it was expected, but they increased approximately linearly.

Additionally, in Fig. 3 the distribution of autocorrelation times of all 128 analyzed 120-s series (32 persons × 4 kinds of signal: EO-AP, EO-LAT, EC-AP and EC-LAT) is presented. 38 values of autocorrelation times were in the range of $\tau = 0 \div 2$ s, 56 values were in the range of $\tau = 2 \div 8$ s and the rest in the range of $8 \div 26$ s. It means a relatively large between-subject variability of this magnitude.

Calculation DC for different values of W

Mean values of estimated DC for 8 examples of the series (AP/LAT-EO/EC-original/shuffled) are presented in Fig. 4a, b.

In the whole range of *W*s, DCs for original signals were smaller than those for shuffled signals. It confirms the hypothesis that posturographic signal possess a deterministic component. DC increased with a negative acceleration as *W* increased for both original and shuffled signals. No plateau typical for chaotic signals was observed in these figures, which could be used for proper DC calculations. In Fig. 5 a, b mean differences of DFs and ratios RAs of DCs for original and shuffled signals are plotted as a function of *W*.

One can see that maxima of these functions showed large variability for different kinds of signals. EC-AP possessed a maximum of DF for about 9 s and a maximum of RA for about 4 s. In case of EC-LAT, DF_{max} is about 10 s and RA_{max} is about 8 s. For EO-LAT these values were equal to about 4 s and 1 s. EO-AP possessed a clear maximum neither for DF(*W*) nor for RA(*W*).

Statistical significance of difference between DCs of the original and shuffled signals decreased gradually with increasing *W* for all 4 kinds of signals. The highest significance was observed for $W = 0.25$ s. The differences between DCs were insignificant starting from $W > 8$ s and $W > 16$ s for EC-LAT and EC-AP, respectively.

Additionally, in Fig. 6 the relation DC(*W*) was shown for 8 persons. It is visible that individual relations differ considerably.

The common feature of these curves is an approximately linear increase of DC in the range of *W* of about $0.25 \div 4$ s. Large variability of the data can be visible for *W*s of lengths longer than 4 s. Some of the curves possess plateau for $W > 4$, the others gradually increase in the whole range of *W*.

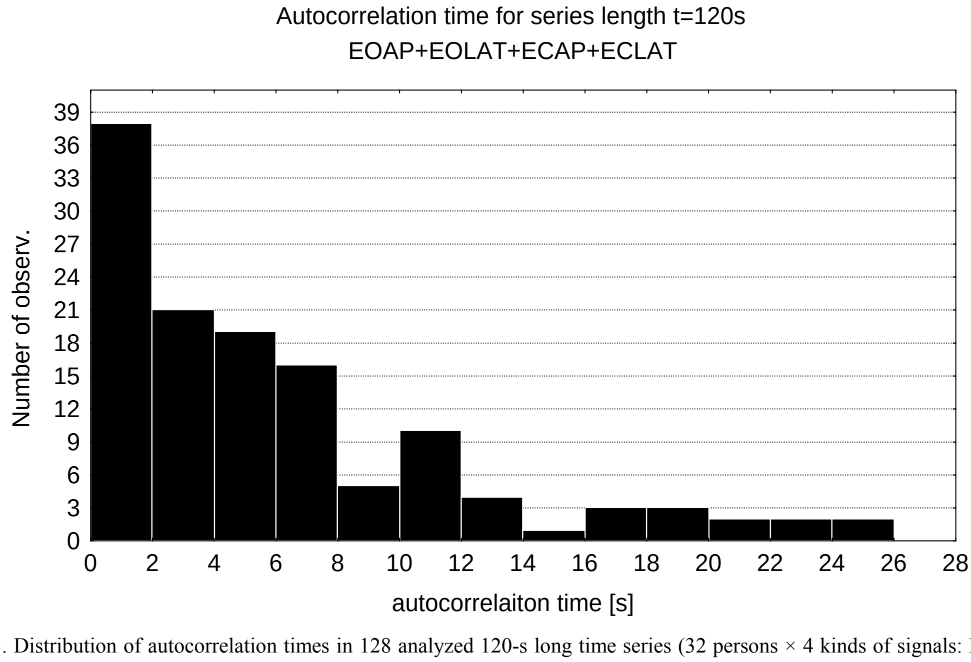
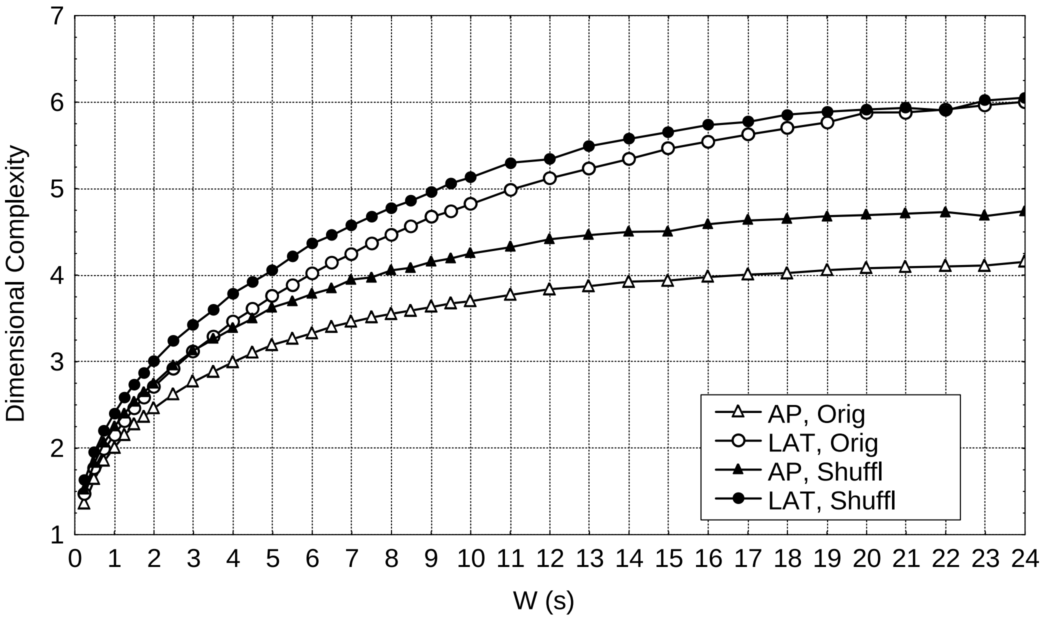


Fig. 3. Distribution of autocorrelation times in 128 analyzed 120-s long time series (32 persons × 4 kinds of signals: EO-AP, EO-LAT, EC-AP and EC-LAT). Large between-subject variability of this variable could be observed.

DC as function of window length W. Eyes Open.



DC as function of window length W. Eyes Closed.

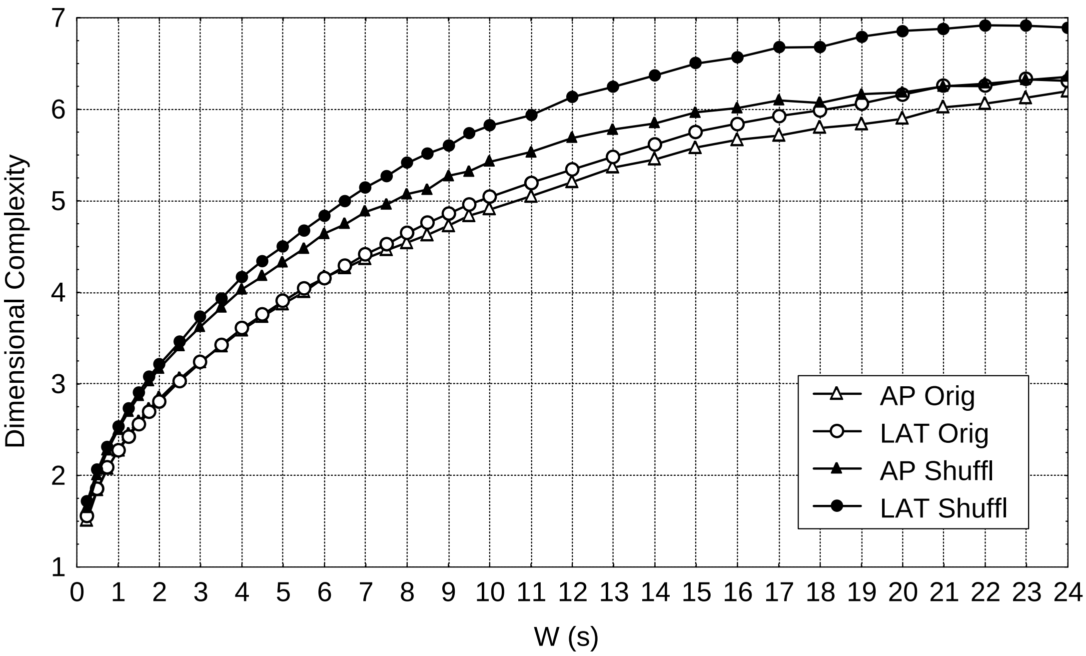


Fig. 4 Dimensional complexity of posturographic signals as a function of window width. (upper plot) Open Eyes, (lower plot) Closed Eyes. *AP* - anterio-posterior direction, *LAT* - lateral direction, *Orig* - original signals, *Shuffl* - shuffled signals. Lack of a plateau and $DC_{orig} < DC_{shuffl}$ in the whole range of *W* is observed.

DISCUSSION

In the present study, we tried to find a proper value of *W* for posturographic signals by means of two different methods: estimation of autocorrelation time and by finding a plateau in the relation between DC and *W*. Additionally, we searched maximal differences and ratios between DCs of original and shuffled signals. The obtained results did not answer unambiguously to the question what a window width should be taken to estimate dimensional complexity of posturographic signal.

4 kinds of signals were analyzed separately: EO-AP, EO-LAT, EC-AP and EC-LAT. Different results obtained for these 4 kinds of signals suggest that their nature differ remarkably.

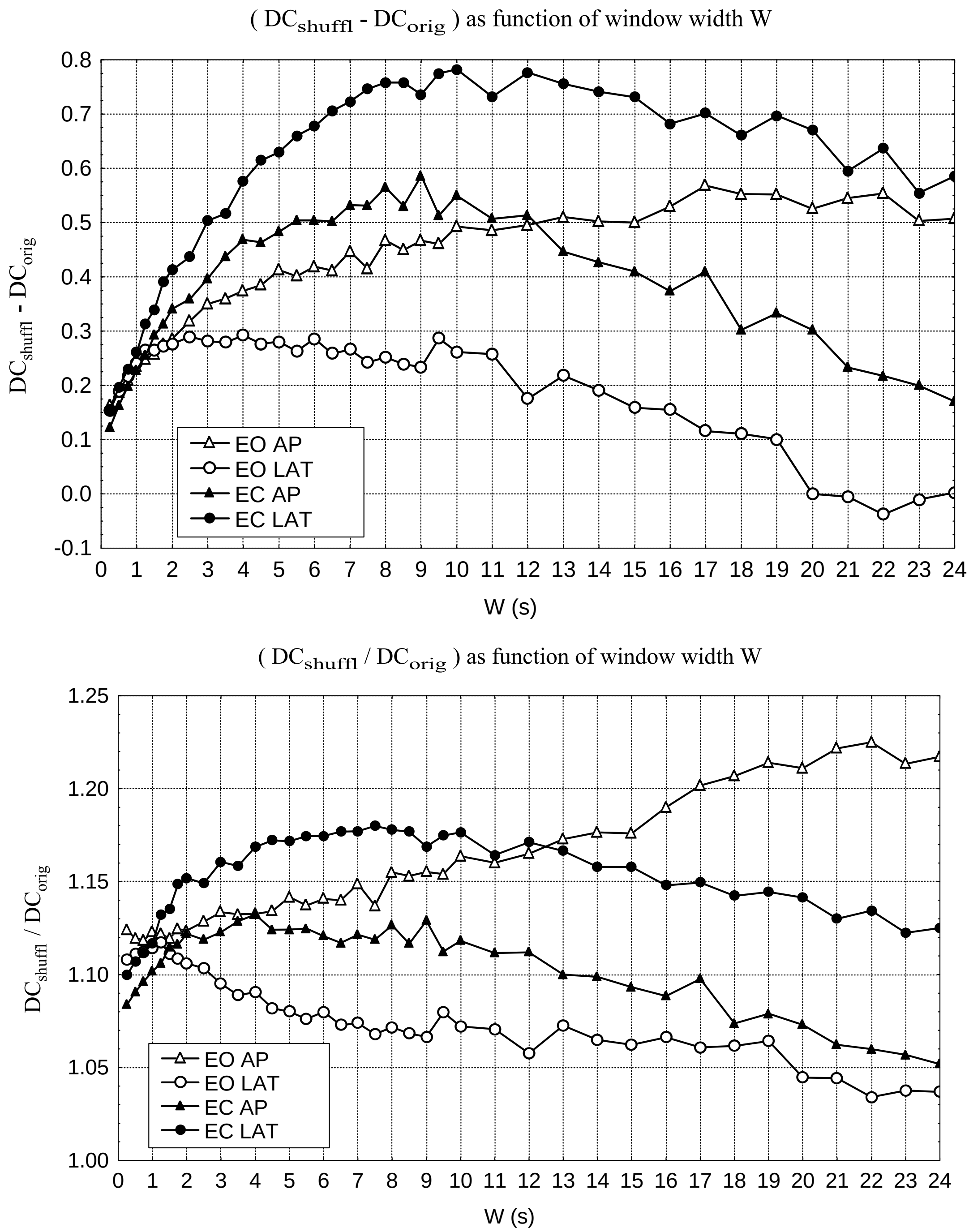


Fig. 5 (a) Differences (DC_{shuffl} - DC_{orig}) between DCs of shuffled and original signals; (b) ratios (DC_{shuffl} / DC_{orig}). Maxima of these relations differ remarkably for separate kinds of signals oscillating mainly about W=1÷10s.

Autocorrelation time

Autocorrelation time of "classical" chaotic signals (e.g. Lorenz attractor) is independent of sequence length. It is commonly assumed that for DC calculations the window width should be approximately $W = \tau \div 4\tau$. Pritchard and Duke proposed a universal value $W = 3\tau$ (Pritchard & Duke, 1992) while Rapp *et al.* claimed that the optimal empirically determined W for Lorenz attractor is $W = 3.33\tau$ (Rapp, Bashore, Martinerie, Albano, Zimmerman & Mees, 1989).

Autocorrelation times of posturographic sequences are approximately linearly related to their lengths. Such increase of autocorrelation time is typical for signals with $f^{-\alpha}$-like spectrum and posturographic signal belongs to them (Michalak & Jaśkowski, 2002). Theiler has proven that in case of signals with $f^{-\alpha}$-like spectrum, autocorrelation time can scale with the size of the data set, being practically as long as about 10-30% of sequence length (Theiler, 1991). So, when a sequence length becomes longer and longer, autocorrelation time increases linearly. If the sequence length becomes longer, new Fourier components of lower and lower frequencies come to analysis. These components have larger and larger amplitudes and, thus, influence autocorrelation time to the largest extent.

As shown in Fig. 3, the highest observed autocorrelation time was about $\tau \approx 25$ s. This is about 20% of the sequence length (120 s). Therefore, one can assume that, regarding the $f^{-\alpha}$-like spectrum, the results were as expected. Unfortunately, because of it, they cannot be a good estimator of window width for DC calculations.

It should be noted that these signals with such a spectrum can have a fractal structure, i.e. structure looking similarly irrespectively of the scale on which it is observed. Examples of such fractal signals one can find in Schreiber (1999). Variance, mean or autocorrelation time of such signals can be linearly related to the sequence length. An interesting question arises, which should be answered in future research, whether or not posturographic signal possess fractal properties?

Calculation DC for different values of W

High significance of differences of DCs between original and shuffled signals for small *W* less than about 4 s suggests that proper *W* should be of about this value. Unfortunately, a plateau segment was not observed in this range. On the other hand, maxima of DFs and RAs were observed mainly in the range of $W = 1 \div 10$ s suggesting that *W* should not exceed 10 s. Decrease of these factors for $W >$ 10 s suggests that *W* from this range is too long. Ends of so long vectors (being the points in the state space) are probably weakly correlated and the attractor's structure becomes invisible.

The lack of plateau in the plot DC(*W*), on the one hand, and high significance of DCs' differences, on the other hand, can be due to some reasons. One reason could be the nonstationarity of posturographic signal caused by irregular respiratory movements, the tiredness increasing during the recording period, both physical (muscular) and mental. This could disturb the results, especially in case of long recording times.

The second reason of the lack of a plateau could be an influence of a stochastic component. A remarkable contribution of both deterministic and stochastic components into posturographic signal was postulated by Riley *et al.* (Riley, Balasubramaniam & Turvey, 1999). They developed a method of nonlinear analysis called recurrence quantification analysis (RQA). Results obtained by these methods showed that posturographic signals possess some deterministic dynamics observed against the background of the stochastic activity. Of interest is that their results suggested some kind of nonstationarity in posturographic signals under all registration conditions.

A reason for high variability of the relation DC(*W*) for high values of *W* ($W > 4$ s) might be due to $f^{-\alpha}$-like spectrum of posturographic signal. Michalak and Jaśkowski (2002) found that α coefficients varied in the range of $0.9 \div 1.4$. Theiler (1991) has proven that DC of pink noise depends on α and for $\alpha \in \langle 1,3 \rangle$ it is equal to a finite value of $d_2 = 2/(\alpha\text{-}1)$. For $\alpha = 1.4$, d_2 is equal to 5 and for $\alpha = 1$ d_2 is equal to infinity. This could explain why some plots of DC(*W*) were leveled off for $W > 4$ s corresponding to $\alpha \approx 1.2 \div 1.4$. Sequences with spectrum for which $\alpha \approx 0.9 \div 1.1$ would have no plateau in this range.

Comparison of the α coefficients and DCs for separate signals (AP/LAT - EO/EC - original/shuffled) calculated for some values of window widths (W = 1, 4, 8 and 24 s) forced us to consider the above presented claim with some caution. In most cases there was no significant correlation between α and DC for various kinds of signals. Some significant correlations were observed especially for shuffled signals. This fact confirms the relatively weak effect of α coefficient on calculated DC. It must be kept in mind, however, that spectra of posturographic signals are $f^{-\alpha}$-like only approximately. Visual inspection of analyzed spectra in log-log scale suggests that sometimes the initial part of spectrum representing the lowest frequencies decreases slower than the further part representing the higher frequencies. This is probably due to weak correlation between α and DC but this problem requires further investigations.

A next problem which was only tentatively addressed in this paper is that oscillations representing by the lowest frequencies, lower than, say, 0.5 Hz, have relatively large amplitudes of about 10,000 units. The question is to what degree they mask the oscillations of higher frequencies and of much smaller amplitudes? We suppose that these higher frequencies can probably play an important role in the process of maintenance of body balance. In such signals, possible regularities for high frequencies with small amplitudes can be nearly invisible for classical algorithms estimating DC because large oscillations of the lowest frequencies significantly dominate over them.

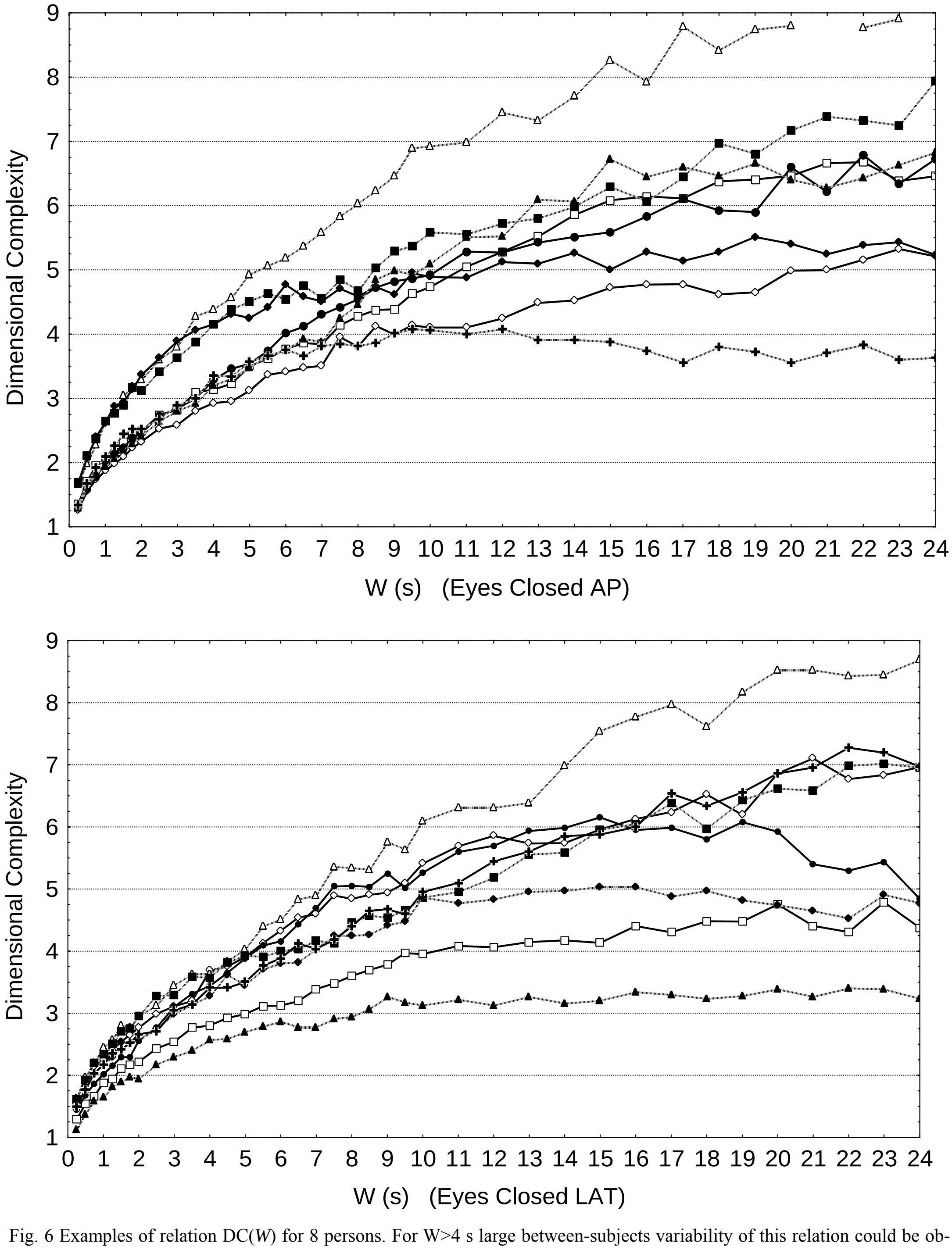


Fig. 6 Examples of relation DC(*W*) for 8 persons. For W>4 s large between-subjects variability of this relation could be observed.

Low frequencies can be considered as reflecting the movement of center of body mass over the ground plane. These movements are slow, with large amplitude, often accidental. Sometimes one can see that the center of participant's mass moves during the recording skipping-wise. For instance, during the first 20 s participant moves slightly forwards, the next 40 s backwards and finally forwards again. The examples of such signals are presented in Fig. 7.

This kind of behavior is probably due to the drift of the center of body mass over the ground or due to tiredness of muscle groups loaded at the moment. For example, if the center of mass moves forwards, the tension of flexors of foot increases and the tension of its extensors decreases. This

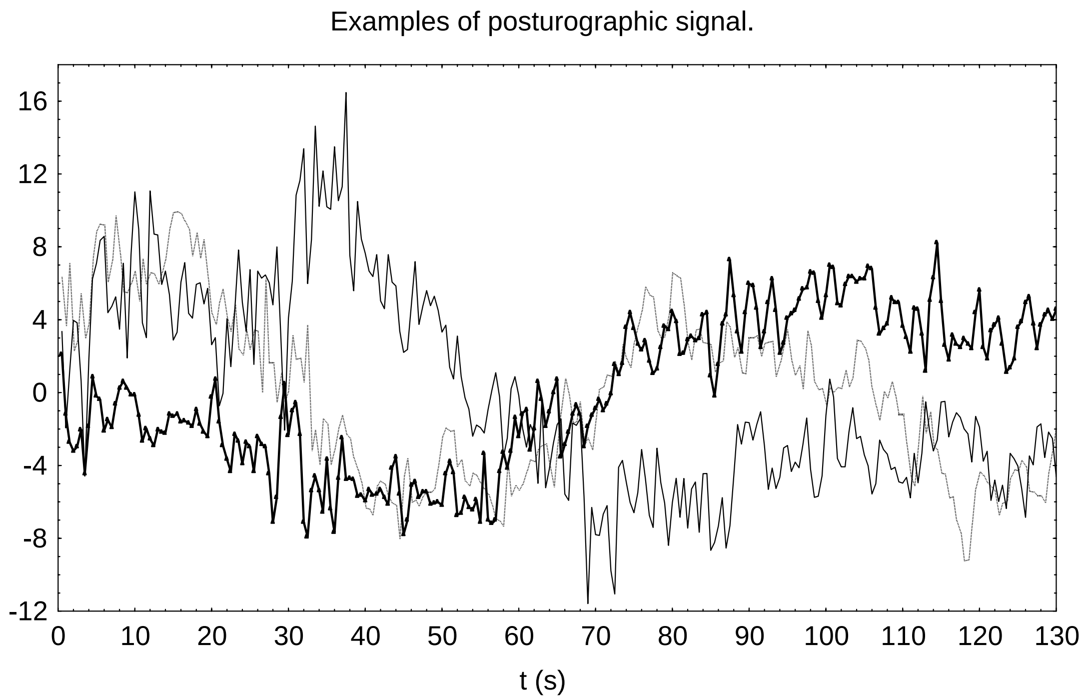


Fig. 7. 3 examples of posturographic signal presenting the drift of center of mass over the ground. In the background of these slow movements of large amplitudes, small high frequency oscillations are observed which reflect proper postural reflexes.

could contribute to the nonstationarity of posturographic signals, the problem discussed above, especially when long recording periods is used. During these long periods small background oscillations of 0.5-20 Hz are visible which ensure proper body balance. The source of these oscillations should be noticed in unconscious postural reflexes.

Therefore, an important question arises: to what extent the calculated DC reflects the slow movement of center of mass (low frequencies, large amplitudes) and to what extent the efficiency of postural reflexes (high frequencies, small amplitudes)? It seems to be purposeful to perform the DC calculations after filtering off the low frequencies below ca. 0.5 Hz to get signal reflecting the activity of postural reflexes only. The proposed limit of frequency (0.5 Hz) was calculated theoretically from the formula: $f = 1/(2\pi\sqrt{l/g})$, where f is the frequency of mathematical pendulum. Putting the length of l = 0.7 ÷ 0.9 m as the distance of human's center of mass to the ground, we obtain the frequency of oscillations ca. 0.5 ÷ 0.6 Hz. Thus higher frequencies observed in the spectrum would probably reflect postural reflexes, not connected with the movement of center of mass. Their efficiency seems to be most important in the maintenance of body balance.

DC calculations of filtered data will be published in a separate paper. According to some preliminary calculations, autocorrelation times were in the range of 180 ÷ 220 ms. This suggests that attention should be paid to the range of W = 0.2 ÷ 1 s which probably carries the information about postural reflexes (carried by frequencies > 0.5 Hz). DC calculated for the range of W = 1 ÷ 10 s would be probably connected with slow movements of center of mass over the ground.

Another problem is a question if posturographic signal is a superposition of some elementary signals originated from different levels of the body (ankles – knees – hips – loins – neck – head). The systems regulating the body equilibrium on these elementary levels are probably simpler in their structure than that generating the resultant posturographic signal. Thus, it is likely that the difference between, say, knees and hips would not possess $f^{-\alpha}$-like kind of spectrum. Therefore autocorrelation time should not increase with time and estimation of proper W would not be as problematic as for posturographic signal. To answer this question one needs a more complex registration system which enables to monitor the movement of separate parts of the body.

A new possibility of posturographic investigation would be to analyze the velocity of movement of COP (V_{COP}) instead COP itself, i.e. the first derivative rather than the original of posturographic signal. V_{COP} should be especially affected by the fast and short postural reflexes which should be disturbed only negligibly by slow movement of center of mass of the body. The problem is that creating the V_{COP} signals would force to filter off the high frequency noise before calculations, because with increasing sampling frequency, noise contamination of V_{COP}-s will be increasingly larger. So, filtering parameters have to be established before performing these calculations.

CONCLUSIONS

The results presented in this article suggest that it is rather impossible to specify unambiguously the window width for DC calculations of posturographic signal. This means that posturographic signal possess no specific value of DC. It is possible only to find relations between different participant groups using DCs calculated for *ad hoc* specified values of W. These calculated DCs would not be equal to the dimensional complexity of the signal but it is possible that these parameters will significantly distinguish some kinds of postural control disturbances. For further investigations the range of $W = 0 – 10$ s seems to be reasonable.

Nevertheless we argue that posturographic signal is not stochastic in nature but possesses some kind of regularity as indicated by significantly smaller DC of original than stochastic signals. This significant difference was observed in the wide range of W for all 4 kinds of signals (EO/EC-AP/LAT).

The lack of unambiguous value of window width may result from many reasons discussed above: $f^{-\alpha}$-like spectrum properties, nonstationarity of the signal, drift of centre of mass during registration process and superposition of signals generated by separate levels of the body.

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