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Fractional Bloch's Equations approach to magnetic relaxation

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It is the goal of this paper to present general strategy for using fractional operators to model the magnetic relaxation in complex environments revealing time and spacial disorder. Such systems have anomalous temporal and spacial response (non-local interactions and long memory) compared to systems without disorder. The systems having no memory can be modeled by linear differential equations with constant coefficients (exponential relaxation); the differential equations governing the systems with memory are known as Fractional Order Differential Equations (FODE).

The relaxation of the spin system is best described phenomenologically by so-called Bloch's equations, which detail the rate of change of the magnetization M of the spin system. The Ordinary Order Bloch's Equations (OOBE) are a set of macroscopic differential equations of the first order describing the magnetization behavior under influence of static, varying magnetic fields and relaxation. It is assumed that spins relax along the z axis and in the x-y plane at different rates, designated as R_1 and R_2 (R_1=1/T_1,R_2=1/T_2) respectively, but following first order kinetics.

To consider heterogeneity, complex structure, and memory effects in the relaxation process the Ordinary Order Bloch's Equations were generalized to Fractional Order Bloch's Equations (FOBE) through extension of the time derivative to fractional (non-integer) order.

To investigate systematically the influence of "fractionality" (power order of derivative) on the dynamics of the spin system a general approach was proposed. The OOBE and FOBE were successively solved using analytical (Laplace transform), semi-analytical (ADM - Adomian Decomposition Method) and numerical methods (Grűnwald- Letnikov method for FOBE). Solutions of both OOBE and FOBE systems of equations were obtained for various sets of experimental parameters used in spin ½ NMR and EPR spectroscopies. The physical meaning of the fractional relaxation in magnetic resonance is shortly discussed.

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